Lecture 15

Homework

Homework #6

- Yariv and Yeh
  - Problems 10.7, 10.11, 11.2, 11.8
- A p-i-n photodiode is able to convert a pulse of light with 8 x 10^{12} photons into 3 x 10^{12} electrons that contribute to the output photocurrent. Calculate the quantum efficiency η and the responsivity R at λ_0 = 0.83 µm, 1.3 µm and 1.55 µm. Now assume that the photodiode is composed of $In_{0.70}Ga_{0.30}As_{0.64}P_{0.36}$ and that the intrinsic region perpendicular to the incident photons is 1 µm thick. Use Figure 11.15 for absorption as a function of wavelength for different material systems/compositions to estimate the quantum efficiency and responsivity at λ_0 = 1.3 mm.
- A silicon p-i-n photodiode operating with 0dBm input at 0.8 µm has 20MHz bandwidth, 65% quantum efficiency, 1nA dark current and 8pf junction capacitance.
  - (a) Determine the RMS current noise due to shot noise.
  - (b) Determine the SNR due to shot noise.
  - (c) If we require and SNR of 20dB, calculate the minimum received optical power when shot noise is the only noise source.

Laser Phase Noise

- Laser noise comes from
  - Cavity length fluctuations
  - Vibrational and temperature fluctuations
  - Quantum mechanical fluctuations
- The later, quantum mechanical fluctuations, occcurs from the spontaneous emission power being random events that continuously add to the output laser emission field
- Each spontaneous emission event adds a random phase $\Delta\theta$ to the optical (electric) field

$$\Delta\theta = \frac{1}{\sqrt{n}}\cos\phi$$

- After N uncorrelated spontaneous emission events, assuming a uniform distribution of $\Delta\theta$ over $[0,2\pi]$ the total field phase does an angular random walk, averaging over a large number of spontaneous emission events

$$\left\langle \left[\Delta\theta(N)\right]^2 \right\rangle = \left\langle \left(\Delta\theta_{\text{one emission}}\right)^2 \right\rangle N = \frac{1}{\bar{n}}\left\langle \cos^2\theta \right\rangle N$$

Laser Phase Noise

- The total number of spontaneous emissions per second sssuming a gain lineshape g (ν) and spontaneous emission into a spectral bandwidth of $\Delta\nu$ is $(VN_2/t_{spont})\, g(\nu)\, \Delta\nu$.
- If we define the number of spontanous "modes" that participate in laser emission as N_{mode}, then the total number of transitions per second into one mode is given by

$$\frac{N_{spont}}{\text{mode-second}} = \frac{VN_2}{t_{spont}N_{\text{mode}}} g(\nu)\Delta\nu = \left(\frac{N_2}{\Delta N_t}\right)\frac{V\Delta N_t}{t_{spont}N_{\text{mode}}} g(\nu)\Delta\nu$$

$$\Delta N_t = \left(N_2 - N_1\right)\Big|_{\text{Threshold Population Inversion}} = \frac{N_{\text{mode}}t_{spont}}{Vg(\nu)\Delta\nu t_c}$$

- Define the population inversion factor $\mu=N_2/\Delta N_t$, the number of transitions into a single mode at time t is $N(t) = \mu t/t_c$ and the rms phase deviation can be written as over an observation interval T

$$\Delta\theta(T) = \int_0^T \left\langle\left[\Delta\theta(t)\right]^2\right\rangle^{1/2} == \sqrt{\frac{1}{2\bar{n}}\frac{\mu}{t_c}T}$$

Laser Phase Noise

- This shift in phase due to laser noise results in a frequency shift Δω that can be written using the definitions for P_e and the observation interval bandwidth B as

$$P_e = \frac{\bar{n}\hbar\omega_0}{t_c}$$

$$B = \frac{1}{2T}$$

$$\left(\Delta\omega\right)_{rms} = \frac{\Delta\theta(T)}{T} = \sqrt{\frac{1}{2\bar{n}}\frac{\mu}{T}} = \sqrt{\frac{\mu\hbar\omega_0}{P_e t_c^2}B}$$

- Δω can be measured using an instrument whose output is the frequency ω(t) = δθ/dt.

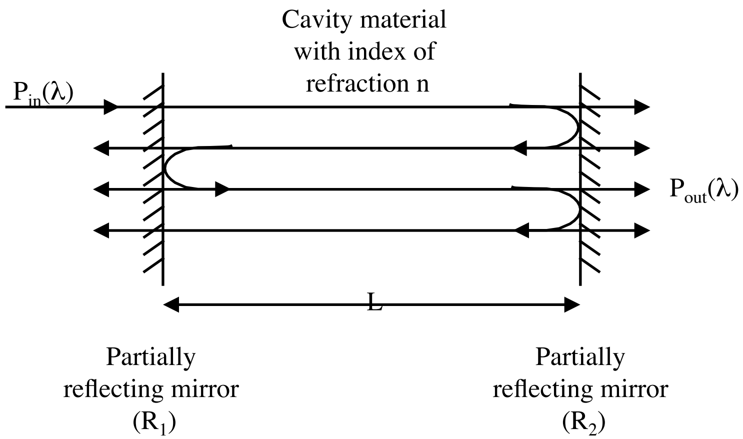
Laser Spectrum

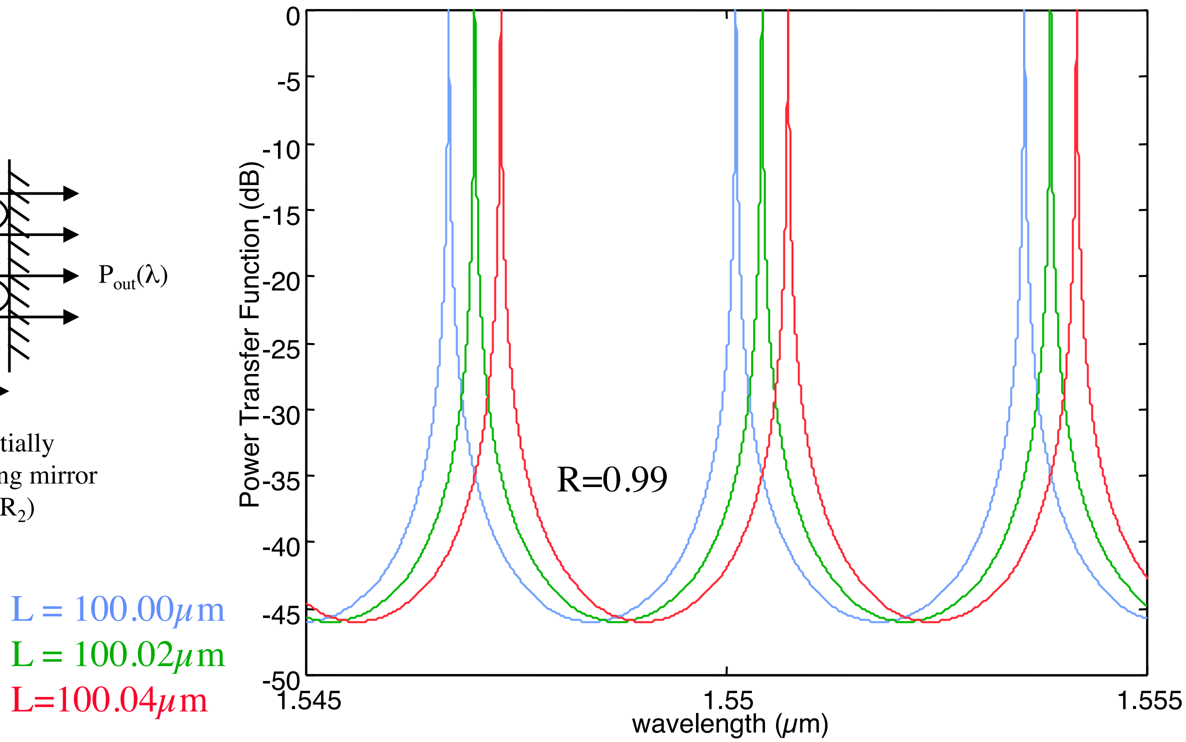
- We can use a variety of instruments to measure the "linewidth" or optical spectrum of a laser.
- For example the length of a Fabry-Perot can be scanned, and if it has high enough resolution, its output will be the laser output power spectrum.
- The output power spectrum will depend on the phase noise we just discussed. Clearly this should be somewhat like the outcome of our circuit equivalent model developed previously.
- Using the autocorrelation function and its relation to the power spectrum via the Wiener-Khintchine Theorem, and write the result (see text for derivation) using the power emitted P and passive resonator linewidth $\Delta\nu$

$$(\Delta\nu)_{laser} = \frac{(\Delta\omega)_{laser}}{2\pi} = \frac{\mu}{4\pi\bar{n}t_c} = \frac{2\pi h\nu_0\left(\Delta\nu_{1/2}\right)^2\mu}{P}$$

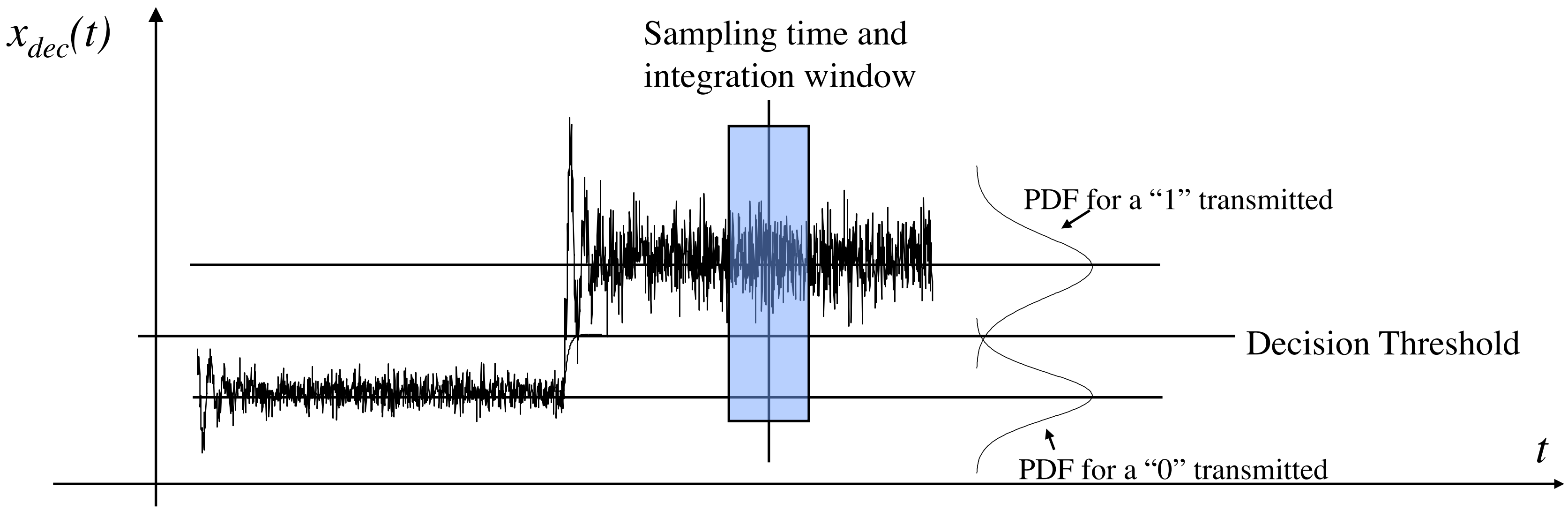
Fabry-Perot Cavities

⇨ The equivalent of an electronic comb filter, but for optical frequencies, the Fabry-Perot (FP) cavity is used for feedback in lasers and as optical filters





Data Recovery: DD Receivers

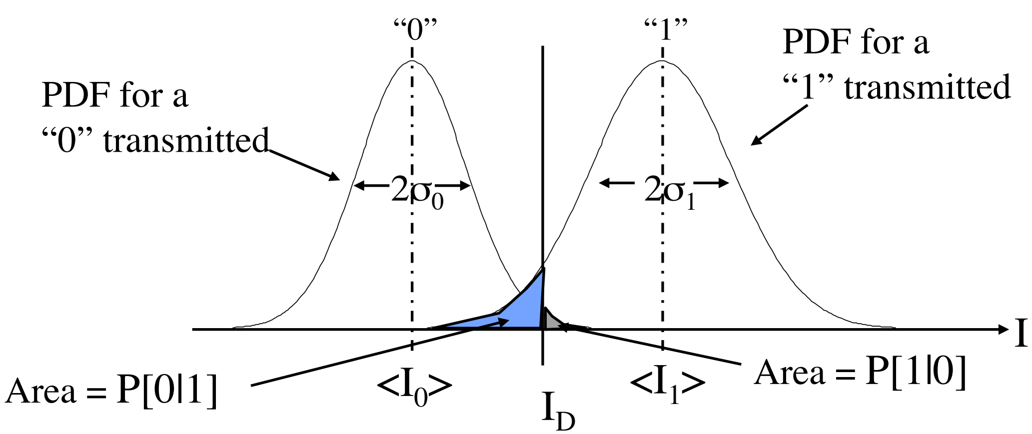


- ➢ The bit error rate (BER) of the system depends from the statistics of the resulting noise on the "1" and "0" levels

Bit Error Rate (BER)

⇒Probability of error = *P[0]P[1|0] + P[1]P[0|1]*

- ⇒*P[0]* = Probability a “0” was transmitted
- ⇒*P[1]* = Probability a “1” was transmitted
- ⇒*P[1|0]* = Probability a “1” is received given that a “0” is transmitted
- ⇒*P[0|1]* = Probability a “0” is received given that a “1” is transmitted



Under the gaussian assumption:

$$P[1|0] = \frac{1}{\sigma_0\sqrt{2\pi}}\int_{I_D}^{\infty} \exp\left\{\frac{\left(\langle I_0\rangle - I\right)^2}{2\sigma_0^2}\right\} dI$$

$$P[0|1] = \frac{1}{\sigma 1\sqrt{2\pi}}\int_{\infty}^{I_D} \exp\left\{\frac{\left(\langle I_1\rangle - I\right)^2}{2\sigma_1^2}\right\} dI$$

BER and Q-Factor

Substituting

$$Q_0 = \frac{I_D - \langle I_0 \rangle}{\sigma_0}$$

$$Q_1 = \frac{I_D - \langle I_1 \rangle}{\sigma_1}$$

$$P[1|0] = \frac{1}{\sqrt{2\pi}} \int_{Q_0}^{\infty} \exp\left\{-\frac{I^2}{2}\right\} dI$$

$$P[0|1] = \frac{1}{\sqrt{2\pi}} \int_{Q1}^{\infty} \exp\left\{-\frac{I^2}{2}\right\} dI$$

The "near" optimum decision threshold is
$$I_D = \frac{\sigma_0 \langle I_1 \rangle + \sigma_1 \langle I_0 \rangle}{\sigma_0 + \sigma_1}$$

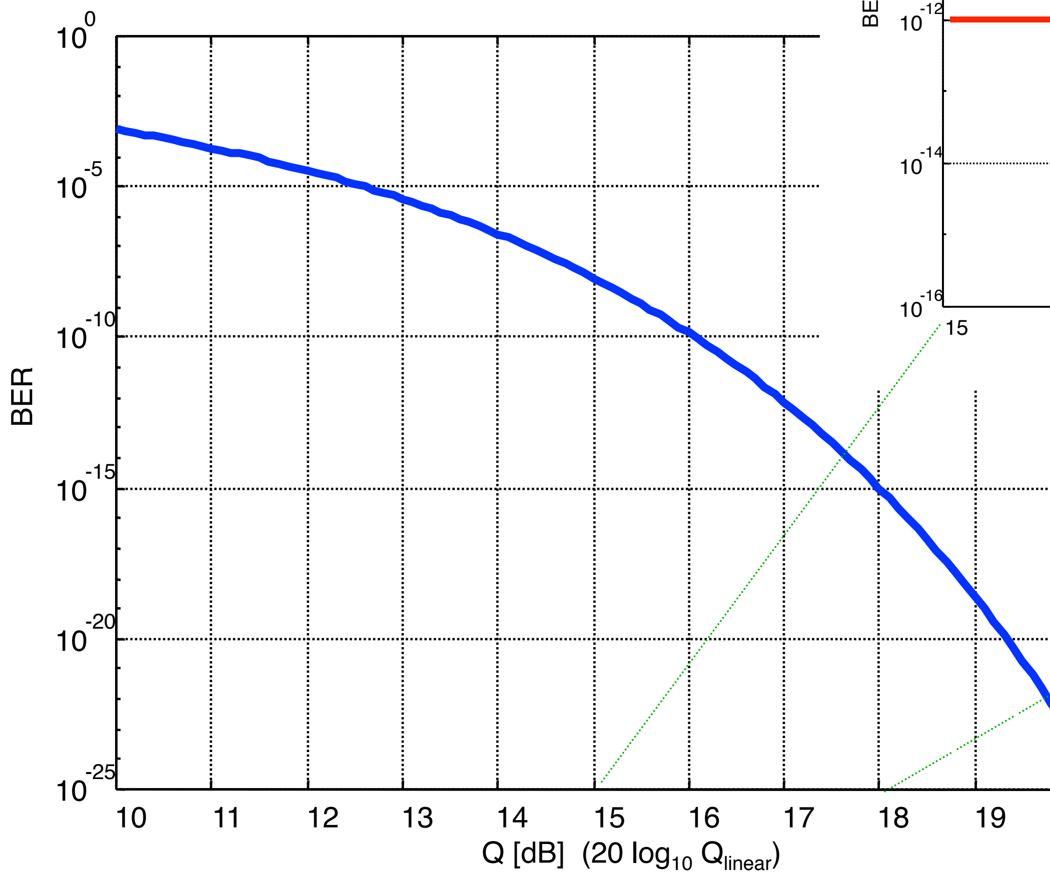
Defining the Q factor
$$\boxed{Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0}}$$

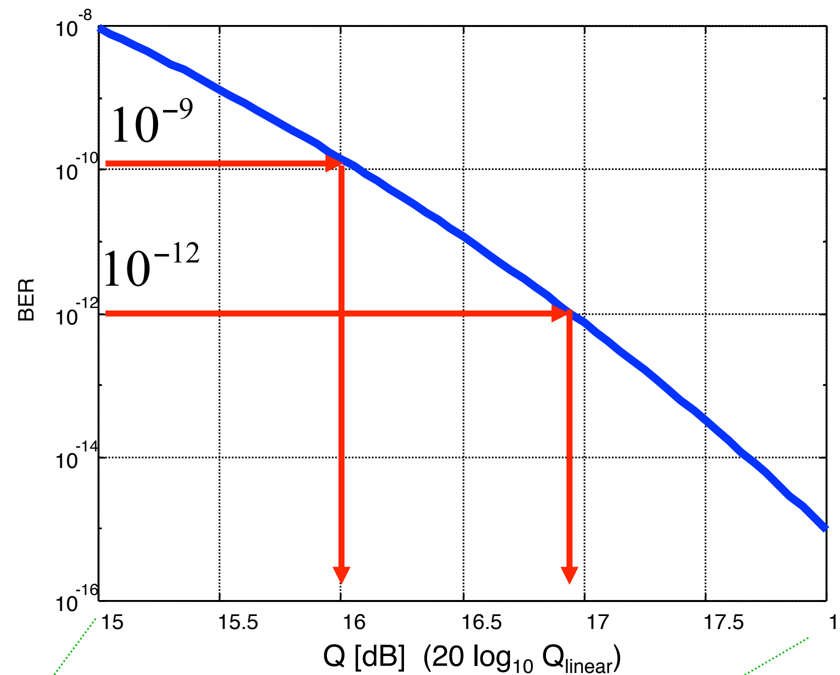
- BER is the most important performance indicator of a receiver
- Q-factor is a good indicator

The bit error rate (BER) assuming Gaussian noise can be written as

$$\boxed{BER \cong \frac{1}{2} erfc\left(\frac{Q}{\sqrt{2}}\right) \approx \frac{\exp\left(-Q^2/2\right)}{Q\sqrt{2\pi}}}$$

BER vs. Q-Factor



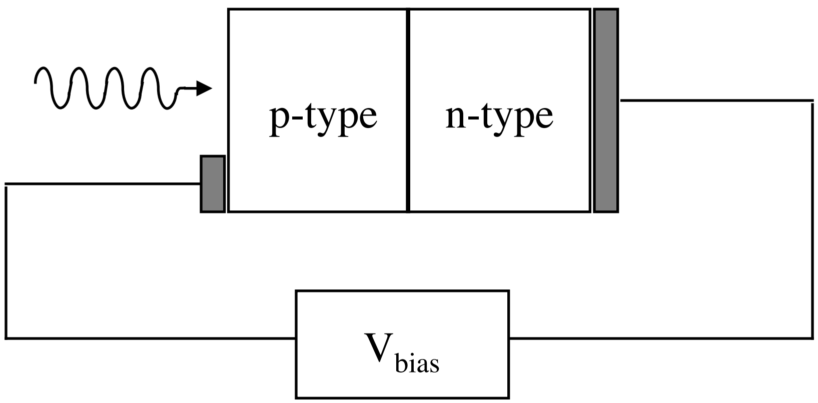


Important values:
- Q~16 dB -> BER=10^{-9}
- Q~17.0 dB-> BER=10^{-12}
- slope: 2 dpades per dB around BER=10^{-9} (slope increases when increasing reference BER)

Detection of Optical Signals

- Thermal: Temperature change with photon absorption
  - Thermoelectric
  - Pyromagnetic
  - Pyroelectric
  - Liquid crystals
  - Bolometers
- Wave Interaction: Exchange energy between waves at different frequencies
  - Parametric down-conversion
  - Parametric up-conversion
  - Parametric amplification
- Photon Effects: Generation of photocarriers from photon absorption
  - Photoconductors
  - Photoemissive
  - Photovoltaics

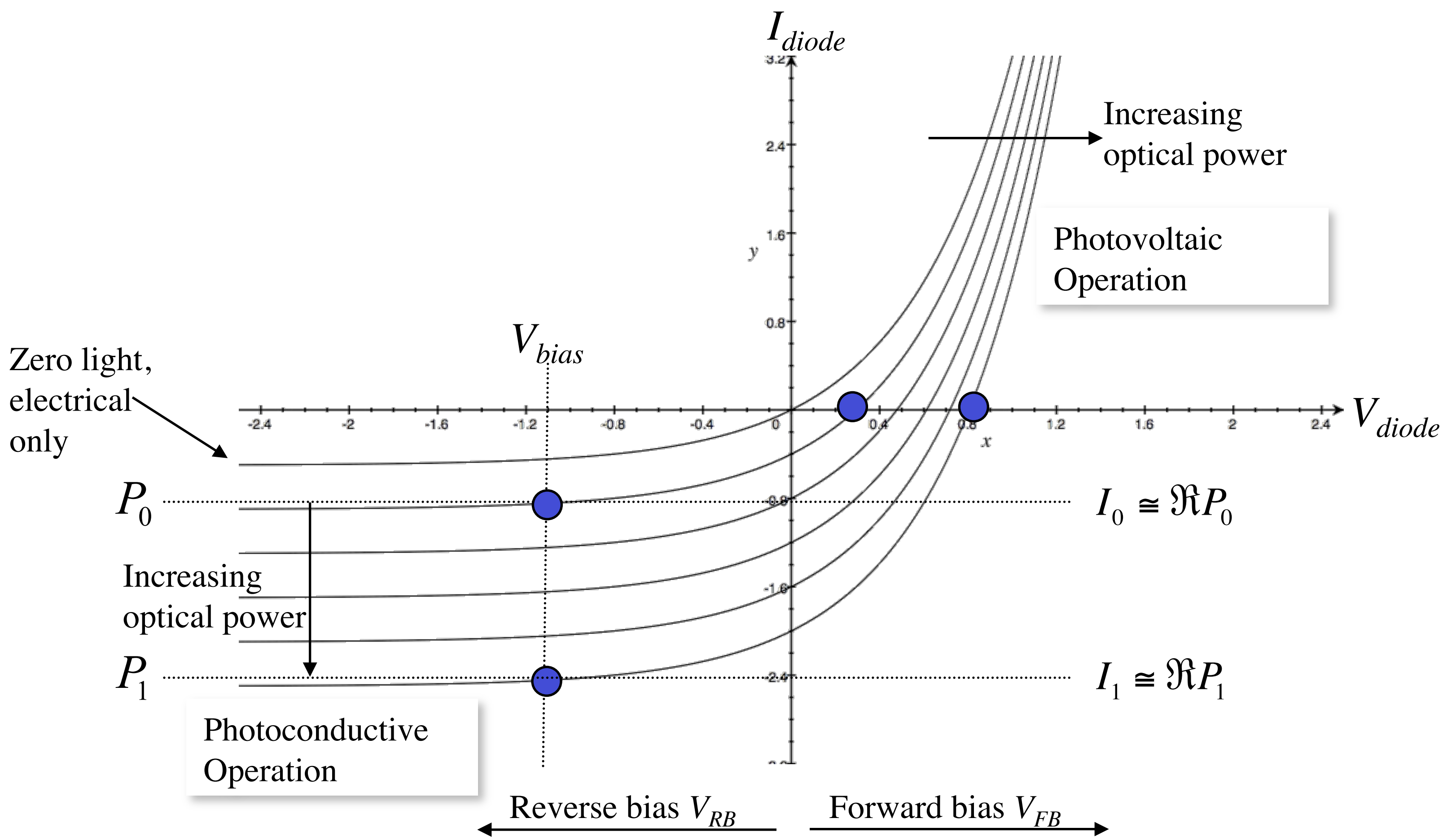
p-n Junction Photodiode Equation



$$I = (I_s)[\exp^{qV_{bias}/K_B T} - 1] - I_{photo}$$
$$= I_{dark} - I_{photo}$$

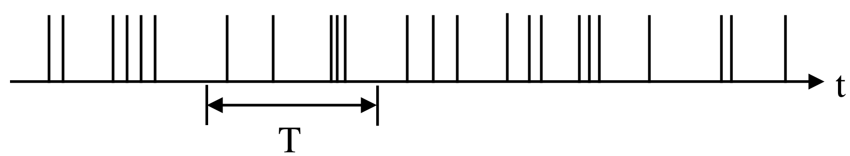
- I_{dark} = is the current that occurs with zero optical input
- $I_s = I_{th}$ is the thermal or saturation current that occurs in normal (non-illuminated) diode operating mode
- I_{photo} is photo-generated current = $\frac{\eta q}{h\nu} P_{rcvd}$
- q is the electron charge
- V_{bias} is applied bias voltage (positive = forward, negative=reverse)
- K_B is Boltzman's constant
- T is temperature (usually in Kelvin, depending on units of K_B)

p-n Junction Photodiode Regions of Operation



Photon Statistics

- Photon sources can in general be characterized as coherent or incoherent†
  - Coherent: Probability that a photon is generated at time t_0 is mutually independent of probability of photons generated at other times (Markov Process)
    - Poisson Process: Probability of finding *n* photons in time interval *T*
      - Bunching is a trait of the Poisson process
      - Interarrival time is decaying exponentially distributed



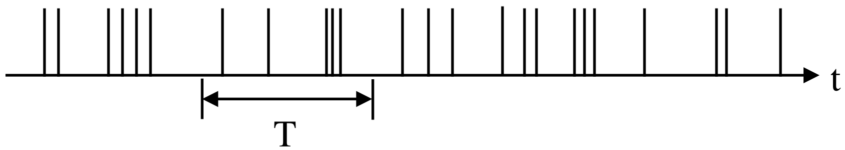
$$\boxed{P(n \mid T) = \frac{(rT)^n e^{-rT}}{n!}}$$

† Can also be a combination of these two types -> partially coherent

Where :
P(n|T) is probability of finding n photons in time interval T
R is mean photon arrival rate (photons/second)

Photon Statistics (II)

- Narrowband Thermal (Gaussian):
  - Bose-Einstein Process: Probability of finding *n* photons in time interval *T*

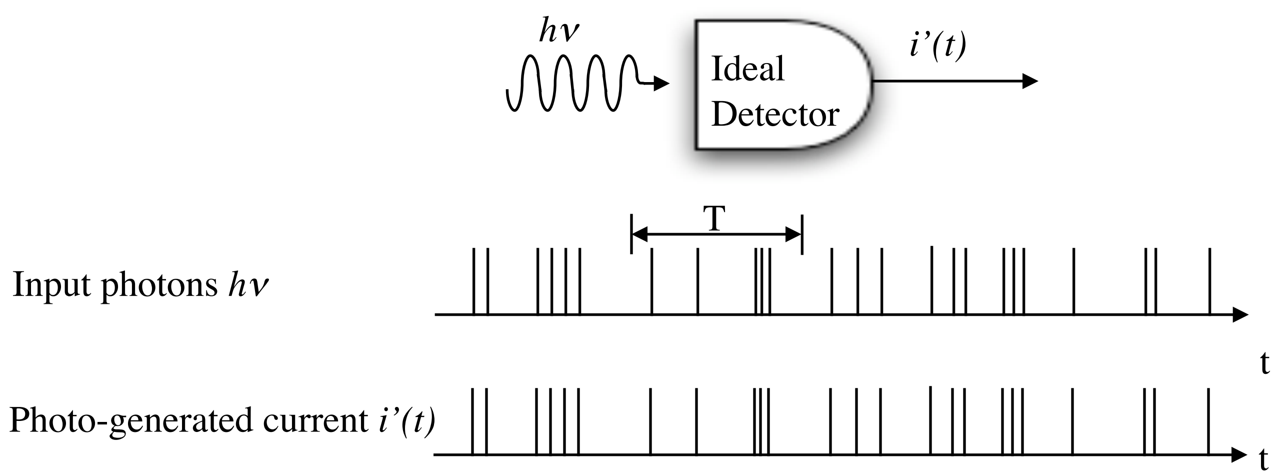


$$\boxed{P(n) = \left(\frac{1}{1+n_b}\right)\left(\frac{n_b}{1+n_b}\right)^n}$$

Where :
P(n) = probability of finding n photons given
n_b = mean number photons from incoherent source = $N_0/h\nu_0$
N_0 = spectral density of source = P_{opt}/B_0
P_{opt} = total optical power from source
B_0 = source optical bandwidth
T = observation time ≤ $1/B_0$

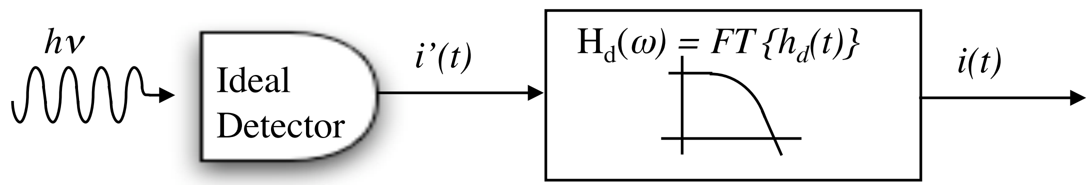
Detecting Photons (1)

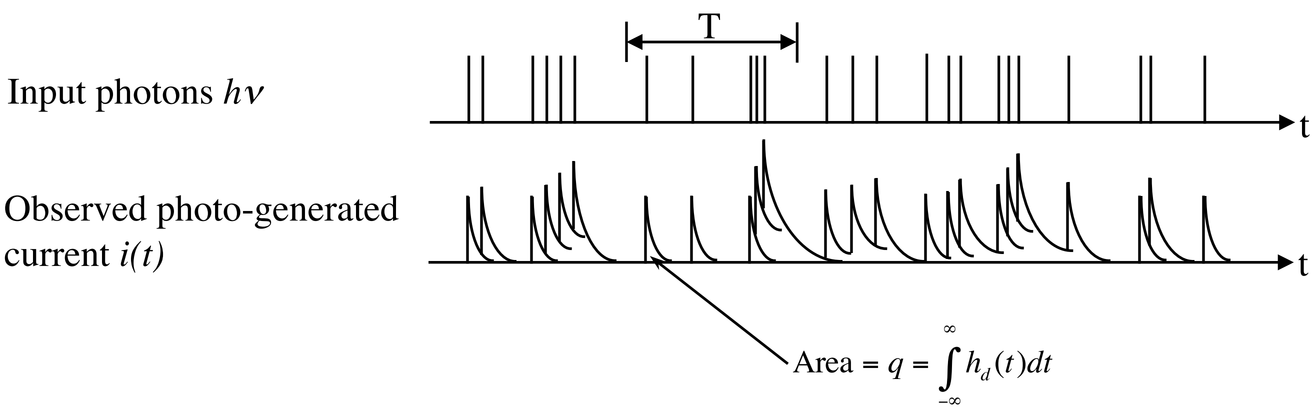
- Any material that can respond to single photons can be used to count photons
- Ideal Detector
  - Generation of a electron-hole pair per absorbed photon results in an instantaneous current pulse



Detecting Photons (2)

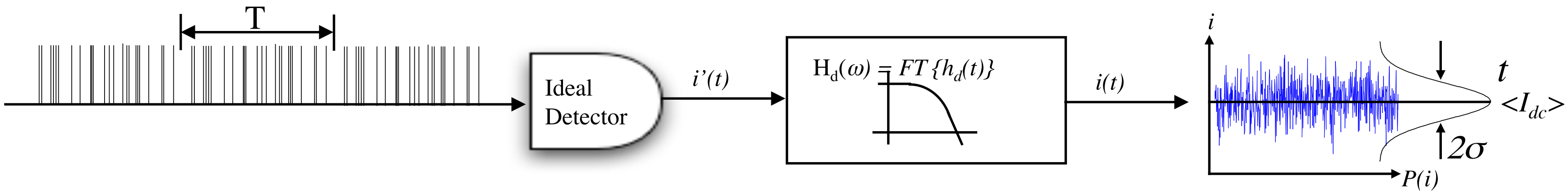
- Real Detector
  - Has an inherent "impulse response," $h_d(t)$, due to built in resistance and/or capacitance.
  - Can be modeled as an RC filter with low pass response





Detecting Photons (3)

- As the average photon rate increases, the observed photo-current starts smoothing out, with a variance around the mean (average) count that is based on the statistics (which tends to Gaussian for large photon arrival rate)
- *P(i)* is the probability function of measuring the current at a certain value at time t.



Detecting Photons (4)

- The detector output current $i(t)$ can be modeled as a discrete "filtered Poisson" process

$$i(t) = \sum_{j=1}^{N} h_d(t - \tau_j)$$

  - Where $h_d(t)$ is PD impulse response, N is total number e-h pairs generated, τ_j is the random time the j^{th} photocarrier is generated.

- Define: Quantum Efficiency (QE), unitless, as

$$\eta = \frac{\text{number of photocarriers produced}}{\text{number of incident photons}}, 0 \leq \eta \leq 1$$

- Define: Time varying photon rate parameter ($\lambda(t)$) in units of photocarriers/second as

$$\lambda(t) = \frac{\eta}{h\upsilon} P_{recvd}(t)$$

Detecting Photons (5)

- The power incident on a photodetector of area A, in units of Watts, is

$$P(t) = \int_A I(\vec{p},t)\,dA$$

  - where the instantaneous optical intensity at an observation point p $\vec{p}$ is given by

$$I(\vec{p},t) = \frac{1}{Z_0}\left|E(\vec{p},t)\right|^2$$

- The time varying photon rate parameter *λ(t)* can then be written in terms of *P(t)*

$$\lambda(t) = \frac{\eta}{h\nu}\frac{\left|E(t)\right|^2}{Z_0}$$

Detecting Photons (6)

- If we consider an observation interval, over which we are going to average our photon count over
  - This can be due either to the inherent bandwidth of the detector or (as we will see later) on purpose to match the receiver bandwidth to the data bit rate
- Then the number of photocarriers generated over the interval T counted at the j^{th} observation interval

$$N_j = \int_0^T \lambda_j(\tau)d\tau$$

- Assuming a coherent source, the ***conditional inhomogeneous Poisson process*** describes this photon count during the j^{th} observation interval

$$P(N_j = N) = \frac{\left(\int_0^T \lambda_j(\tau)d\tau\right)^N}{N!} e^{\left(-\int_0^T \lambda_j(\tau)d\tau\right)}$$

Detecting Photons (7)

- If we assume a constant rate parameter over the time interval T (independent of *j*), then the photo-generated current can be written as

$$i(t) = \lambda(t) q$$

$$\lambda(t) = \frac{N}{T}$$

- Then the photocurrent produced by the photodetector can be written in Amperes, assuming the observation time is normalized to one second

$$\begin{aligned} i(t) = \lambda(t) q &= \frac{\eta q}{h\nu} P_{rcvd}(t) \\ &= \Re P_{rcvd}(t) \end{aligned}$$

  - Where we have defined the detector responsivity as

$$\Re = \frac{\eta q}{h\nu}$$